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LETTER TO THE EDITOR

New approach to the theory of magnetic monopoles

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Abstract. A P, T-invariant theory of massive dually charged particles is proposed. This theory is based on the idea that a particle with multi-spin described by the Dirac-like equation is a magnetically charged particle. The concept of magnetic charge has, in this approach, a pure quantum mechanical nature.

It is well known that the Dirac–Schwinger theory of magnetic monopoles is non-invariant under space (P) and time (T) inversions. The description of magnetically charged particles with spin $\frac{1}{2}$ by the current $j_\mu^5 = \mathrm{i}g\bar{\Psi}\gamma_\mu\gamma_5\Psi$ leads us to the T-non-invariant theory, and the mass origin problem arises (Salam 1966, Taylor 1967).

The aim of our Letter is to construct a P, T-invariant theory of massive dually charged particles with unusual spin properties.

The Dirac-like equation takes on the form (Borgardt 1953)

$$(\Gamma_\mu \, \partial_\mu + m)\Psi(x) = 0, \tag{1}$$

where the 16-component wavefunction $\Psi(x)$ is transformed under superposition of the irreducible representation (IR) $(0,0)\oplus(0,0)'\oplus[(0,1)\oplus(1,0)]\oplus(\frac{1}{2},\frac{1}{2})\oplus(\frac{1}{2},\frac{1}{2})'$, corresponding to a particle with a single mass state and multi-spin states 0, 1 (Durand 1975). Here, the IR of the proper Lorentz group corresponds to an invariant, a pseudo-invariant, an antisymmetrical tensor, a four-vector and a pseudo-four-vector respectively.

The generators of the proper Lorentz group in the IR superposition considered have the form (Durand 1975)

$$J_{\mu\nu} = \tfrac{1}{4}(\Gamma_\mu\Gamma_\nu - \Gamma_\nu\Gamma_\mu + \check{\Gamma}_\mu\check{\Gamma}_\nu - \check{\Gamma}_\nu\check{\Gamma}_\mu), \tag{2}$$

where the matrices Γ_μ, $\check{\Gamma}_\nu$ obey the Dirac conditions and

$$\Gamma_\mu\check{\Gamma}_\nu = \check{\Gamma}_\nu\Gamma_\mu. \tag{3}$$

Equation (1) comes from the Lagrangian

$$\mathscr{L}_0 = -\tfrac{1}{2}[\bar{\Psi}(\Gamma_\mu \, \partial_\mu + m)\Psi - \bar{\Psi}(\Gamma_\mu \, \overleftarrow{\partial}_\mu - m)\Psi], \tag{4}$$

where $\bar{\Psi} = \Psi^+\Gamma_4\check{\Gamma}_4$.

The operations of the charge conjugation (C), space (P) and time (T) inversions are defined as follows:

$$\begin{aligned} &C: \Psi \to \eta_c\Gamma_2\Gamma_4\check{\Gamma}_2\check{\Gamma}_4\bar{\Psi}^{\mathrm{T}}, \\ &P: \Psi \to \eta_p\Gamma_4\check{\Gamma}_4\Psi, \\ &T: \Psi \to \eta_t\Gamma_3\Gamma_1\check{\Gamma}_3\check{\Gamma}_1\Psi^*. \end{aligned} \tag{5}$$

We have conserved currents $J_\mu = \mathrm{i}\bar{\Psi}\Gamma_\mu\Psi$ and $K_\mu = \bar{\Psi}\Gamma_\mu\check{\Gamma}_5\Psi$ which are connected with the invariance of the Lagrangian (4) under the transformations

$$\Psi' = \exp(\mathrm{i}I^{(16)}\alpha + \check{\Gamma}_5\beta)\Psi. \tag{6}$$

The transformations (6) define the group GL(1, C). Under the P, T transformations J_μ is a polar four-vector and K_μ is the axial four-vector.

Let us now define the Lagrangian

$$\mathscr{L} = \mathscr{L}_0 + \tfrac{1}{4}F^2_{\mu\nu} - \tfrac{1}{2}F_{\mu\nu}(\partial_\mu A_\nu - \partial_\nu A_\mu) + eJ_\mu A_\mu + gK_\mu B_\mu, \tag{7}$$

where

$$B_\mu(x) = \int (\mathrm{d}x')\tilde{F}_{\mu\nu}(x')f_\nu(x'-x), \qquad \partial_\nu f_\nu(y) = \delta(y)$$

and the function f_ν describes the line of singularity of electromagnetic potentials (Schwinger 1966). The variation of the Lagrangian with respect to Ψ, A_μ, $F_{\mu\nu}$ leads to the equations†

$$[\Gamma_\mu(\partial_\mu - \mathrm{i}eA_\mu - g\check{\Gamma}_5 B_\mu) + m]\Psi(x) = 0, \tag{8}$$

$$\partial_\nu F_{\mu\nu} = eJ_\mu, \tag{9a}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \epsilon_{\mu\nu\alpha\beta}\int (\mathrm{d}x')f_\alpha(x-x')gK_\beta(x'). \tag{9b}$$

Equation (9b) is equivalent to the second pair of the Maxwell equations

$$\partial_\nu \tilde{F}_{\mu\nu} = gK_\mu, \tag{9c}$$

where

$$\tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu - \epsilon_{\mu\nu\alpha\beta}\int (\mathrm{d}x')f_\alpha(x-x')eJ_\beta(x').$$

So we construct the P, T-invariant theory of massive dually charged particles, which is the theory of a particle with multi-spin‡.

Following the ideas of Schwinger (1966) we can derive, from the requirement of self-consistency of the theory developed, the charge quantisation condition

$$eg = 2\pi n \qquad (n = 0, \pm1, \pm2, \ldots). \tag{10}$$

Now it is necessary to stress the principal difference from the Dirac–Schwinger theory. In the latter case, when dually charged particles are considered, the charge quantisation condition has the form

$$e_i g_j - g_i e_j = 2\pi n_{ij} \qquad (n_{ij} = 0, \pm1, \pm2, \ldots). \tag{11}$$

Here the indices i, j characterise the different types of particles. Considering one type of a particle, as in our case, no restrictions on the value of electric and magnetic charges follow from condition (11), in contrast with condition (10). Further, in the approach developed we do not use any condition *à la* 'Dirac veto' (Wentzel 1966). These differences are due to the pure quantum mechanical nature of the description of a magnetic charge in our approach.

† The procedure is the direct generalisation of the Schwinger (1966) and Yan (1967) approach.

‡ In Pestov (1978) the concept of magnetic charge is considered from another point of view, on the grounds of the theory of a particle with multi-spin 0, 1.

Note that the Lagrangian (7) is invariant under the transformations

$$\Psi' = \exp(\mathrm{i}I^{(16)}\alpha + \check{\Gamma}_5\beta + \check{\Gamma}_{[\mu}\check{\Gamma}_{\nu]}\omega_{\mu\nu})\Psi, \tag{12}$$

with the following restrictions on the parameters:

$$\alpha^* = \alpha, \qquad \beta^* = \beta, \qquad \omega^*_{mn} = \omega_{mn}, \qquad \omega^*_{m4} = -\omega_{m4}. \tag{13}$$

It corresponds to the group SL(2, C)$\otimes$GL(1, C), which is the subgroup of the GL(4, C) invariance group of equations (1) (Boguch *et al* 1978). Such internal symmetry of the theory of particles with single mass state and multi-spin states 0, 1 is called a 'dyal symmetry' (Strazhev 1978). The dyal symmetry is a new kind of internal symmetry, its generators having a tensorial nature from the point of view of the Lorentz group.

The question arises as to whether we can describe fermions in such an approach. One can interpret the wavefunction $\Psi(x)$ in (1) as a cross-product of two bi-spinors. Further, one can use the case of the cross-product of three bi-spinors to describe a particle with a single mass state and multi-spin states $\frac{1}{2}, \frac{3}{2}$ (Feschbach 1955). In this case one can also identify the axial conserved current with the magnetic current.

The unusual spin behaviour and P, T invariance of the electromagnetic interactions in the presence of magnetically charged particles are the remarkable properties of the proposed theory.

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